MATHEMATICAL MODELING AND ANALYSIS OF VARIOUS FLOW CHARACTERISTICS OF BLOOD THROUGH STENOSED ARTERY

A Synopsis

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1 INTRODUCTION

1.1 Circulatory System

The Circulatory System is an organ system which is responsible for transporting materials throughout the entire body. It transports nutrients, water, and oxygen to the body cells and carries away wastes such as carbon dioxide produced by the body cells.

Basic structures of circulatory system consists of :

1. Heart
2. Blood
3. Blood Vessels [5]

1. *Heart*

The heart is the organ that supplies blood and oxygen to all parts of the body. These muscles produce electrical impulses that cause the heart to contract and then relax, producing a heart-beat. The beating of the heart drives the cardiac cycle which pumps blood to cells and tissues of the body.

2. *Blood*

The blood delivers nutrients to cells and removes wastes that are produced during cellular processes (cellular respiration). Blood is composed of red blood cells, white blood cells, platelets and plasma [3].

a) Red blood cells (erythrocytes) make up to about 45% of blood volume .The primary function of red blood cells is to transport oxygen to body cells and deliver carbon dioxide to the lungs.

b) White blood cells (leukocytes) are the cells which protects the body by fighting against the infections and foreign invaders.

c) Platelets (thrombocytes) are blood cells which are responsible for the clotting of blood to stop bleeding.

d) Plasma is the liquid portion of the blood that makes up to 55% of the blood volume. Plasma is a mixture of water, proteins and dissolved substances. Around 90% of plasma is made of water.

3. *Blood Vessels*

Blood vessels are networks of hollow tubes that transport blood throughout the entire body. Blood travels from the heart via arteries to smaller arterioles, then to capillaries, to venules, to veins and back to the heart. Through this process of circulation, substances such as oxygen, carbon dioxide, nutrients, and wastes are exchanged between the blood and the fluid that surround cells [3].

There are three types of blood vessels:

a) *Arteries* – These are blood vessels that deliver oxygen-rich blood from the heart to the tissues of the body.

b) *Veins* – These are blood vessel that transports blood from various regions of the body to the heart.

c) *Capillaries* – These are extremely small blood vessel located within the tissues which are responsible for the exchange of water and chemicals between the blood.

1.2 Atherosclerosis

It is a disease of the cardiovascular system that involves hardening of the arteries due to the deposition of plaque which can restrict the blood flow.

Hardening of the arteries however happens with aging. With the increase in the age, plaque deposited narrows the arteries and make them stiffer which in turns, affects the flow of blood in the arteries. Since the arteries get narrower, results in the formation of clots, therefore obstructing the passage of blood through the arteries. This blockage starves tissues from blood and oxygen which can result in damage or death of tissues. This is a common cause of heart attack and stroke. High blood cholesterol levels can cause hardening of the arteries at a younger age.

High cholesterol levels are the results of an unhealthy lifestyle. Most commonly is having a diet that is high in fat. Other responsible factors are heavy alcohol use, lack of exercise and being overweight.

Other risks for the hardening of the arteries are:

- Diabetes
- Hereditary problem
- Smoking
- High blood pressure [1]

1.3 Factors Responsible for the Transportation of Blood in the Cardiovascular System

1.3.1 Viscosity of blood

Blood viscosity is a measure of the resistance of blood to flow. It can also be described as the thickness and stickiness of the blood. It is also a key screening test that measures how much friction the blood causes against the walls of the vessel, how hard the heart has to work to pump blood out of the vessel and how much oxygen is supplied to organs and tissues of the body [2].

There are few factors which are involved in blood viscosity. One of which is composition of blood. The more fluid in the blood, the thinner it will be and lesser will be the viscosity. High counts of red blood cells can lead to an increase in the thickness of the blood. Fats that circulate in the blood are also an important factor affecting the viscosity. High concentration of fat makes it thicker and therefore increasing the viscosity. Another contributing factor is temperature. At high temperatures, the blood becomes thinner and therefore viscosity decreases.

1.3.2 Flow Rate of blood

Flow through a blood vessel is determined by two factors: the force that pushes the blood through the vessel and the resistance of the vessel to the blood flow. Flow rate also affects the viscosity and vice versa. At low flow rate, the viscosity of blood increases significantly.

1.3.3 Newtonian and Non-Newtonian Fluid

Newtonian fluids are the fluids which follows Newton's Law of viscosity which says that shear stress is directly proportional to shear rate (velocity gradient).

The constitutive equation for Newtonian fluids is:

$$\tau = \mu e$$

where, τ is shear stress and e is shear rate.

So the ratio of shear stress and the shear rate is a constant which is termed as Coefficient of Viscosity i.e. μ.

Non-Newtonian Fluids are those fluids which do not follow Newton's Law of Viscosity. For these, ratio of shear stress to shear rate is non constant.

There are few important classes of Non-Newtonian fluids which are [9]:

a) *Power law* fluid with constitutive equation:

$$\tau = \mu e^n$$

If $n < 1$ we get a pseudo-plastic power-law fluid and for $n > 1$ we get a dilatant power - law fluid.

b) *Bingham plastic* fluid with constitutive equations:

$$\tau = \mu e + \tau_o \quad , \tau \geq \tau_o$$

$$e = 0 \quad , \tau < \tau_o$$

c) *Herschel-Bulkley* fluid with constitutive equations:

$$\tau = \mu e^n + \tau_o \quad , \tau \geq \tau_o$$

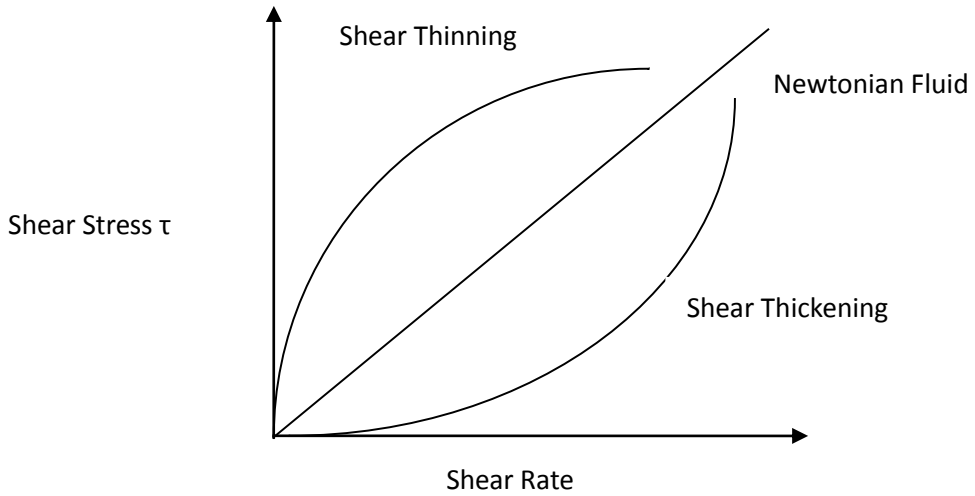
$$e = 0 \quad , \tau < \tau_o$$

If $n < 1$ the fluid is shear thinning fluid, while $n > 1$ produces shear thickening fluid.

d) *Casson-fluid* with constitutive equations:

$$\tau^{1/2} = \mu^{1/2} e^{1/2} + \tau_o^{1/2} \quad , \tau \geq \tau_o$$

$$e = 0 \quad , \tau < \tau_o$$



1.3.4 Laminar and Turbulent Flow

Laminar Flow - It is the flow in which all the particles in the fluid are moving in a straight line. It can also be defined as the flow in which all the particles of a fluid within a layer are moving at the same rate.

Turbulent Flow – The flow in which there is an irregular flow of particles i.e. particles are in the state of chaos. When critical speed of a fluid is reached then the fluid becomes turbulent.

The point at which a fluid becomes turbulent is based on a unit less parameter called *Reynolds Number* given by Osborn Reynolds:

$$\mathrm{Re} = \frac{VL\rho}{\mu}$$

where, Re- Reynolds Number

V- Velocity of flow

L- Diameter of pipe

ρ- Density of fluid

μ -Viscosity of fluid [19]

It is a dimensionless quantity which is directly proportional to velocity of flow, density of fluid, diameter of tube and inversely proportional to viscosity of fluid. For high Reynolds number, the fluid will be less viscous and velocity is high causing intense mixing of the fluid particles making the flow to be turbulent and for small, it is laminar.

1.3.5 Magnetic Field applied on arteries

As blood is an electrically conducting fluid so magnetic field can be applied on it. So if the magnetic field is applied on blood which is flowing in the artery, an electromagnetic force will be produced due to the interaction of current with magnetic field, which is called Lorentz Force.

By Ohm's law, the current density J is given by

$J = \sigma (E + v \times B)$ [18]

where σ is the electrical conductivity, E is the electric field and B is the magnetic field and v is the velocity vector.

The Navier -Stokes equation for blood flow including a Lorentz force **f** is

$$\rho \left(\frac{\partial v}{\partial t} + v \cdot \nabla v \right) = \mathbf{f} - \nabla P + \mu \nabla^2 v \quad [17]$$

where ρ is fluid density, μ is viscosity and P is the pressure.

1.3.6 Elasticity of blood vessels

Blood vessels carry blood from different parts of the body to heart and from heart to various parts of the body, so elasticity property of blood vessels also plays an important role in transportation of blood. As the blood flows through these vessels, they expand to allow for the increase in pressure across the walls and therefore help in the even flow of blood.

1.4 Porous Medium

A porous medium is a substance that contains pores between solid materials through which liquid or gas can pass. Few examples of naturally occurring porous media are sand, soil and some types of stone such as pumice and sandstone. A porous medium can be distinguished from other porous media by its porosity (Measure of void space) or the size of its pores. Materials with low porosity are less permeable and have smaller pores, making it difficult for gas or liquid to pass through them, while materials with high porosity have large pores and are easily permeable.

1.5 Permeability

It is the ability of the porous material to transmit the fluid through it. It only depends on the porosity of the material and independent of other properties of the material.

1.6 Survey of Literature

The literature on this topic is quite vast so we are mentioning few important works done in this field:

Young (1968) analyzed the effect of an axially symmetric, time dependent growth into the lumen of a tube of constant cross section through which a Newtonian fluid is steadily flowing [30].

Shukla et al. (1979) studied the effects of peripheral layer viscosity on physiological characteristics of blood flow through artery with mild stenosis. It has been shown that the resistance to flow and the wall shear decrease as the peripheral layer viscosity decrease [23].

Chaturani et al. (1985) studied the blood flow through stenosed artery considering blood as non-Newtonian fluid obeying Herschel Bulkley equation and the variation of wall shear stress and flow resistance with fluid behavior index, shear-dependent non-linear viscosity and the yield stress has been shown graphically [4].

Rodkiewicz et al. (1988) showed analytically and experimentally that within the scope of a surgery, the effects of variations in the position of the transplant aorta contact point in the transplant length and in the transplant curvature are relatively insignificant regarding mean flow resistance. They concluded that it is not important how the transplant will be situated and that the space convenience should be surgical determining factor. It has been shown that the rate of blood

flow to the kidney may be significantly curtailed if the selected transplant diameter is too small [20].

Misra et al. (2005) developed a mathematical model (Herschel-Bulkley fluid model) for studying the non-Newtonian flow of blood through a stenosed arterial segment. The problem is investigated by a combined use of analytical and numerical techniques and it is noticed that the resistance of flow and the skin-friction increase as the stenosis height increases [14].

Figueroa (2006) discussed that blood velocity and pressure fields in large arteries are greatly influenced by the deformability of the vessel. Wave propagation phenomena in the cardiovascular system can only be described considering wall deformability since blood is usually described as an incompressible fluid. However, computational methods for simulating blood flow in three-dimensional models of arteries have either considered a rigid wall assumption for the vessel or significantly simplified or reduced geometries when modeling blood flow in deformable arteries. Computing blood flow in deformable domains using standard techniques like the ALE method remains a formidable problem for large, realistic anatomic and physiologic models of the cardiovascular system. They have developed a new method termed the Coupled-Momentum Method for Fluid-Solid Interaction to simulate blood flow in three-dimensional deformable models of arteries. In this method, the effect of the vessel wall boundary is added in a monolithic way to the fluid equations using a shear-enhanced membrane model for the wall, resulting in a remarkably robust scheme and have verified the method by comparing it against Womersley's analytical solution for pulsatile flow in a straight cylindrical elastic vessel, and have obtained excellent agreement between the numerical and analytical solutions [6].

Vignon- Clementel (2006) discussed that flow and pressure waves emanate from the heart and travel through the major arteries where they are damped, dispersed and reflected due to changes in vessel caliber, tissue properties, and branch points. As a consequence of which solutions to the governing equations of blood flow in the large arteries are highly dependent on the outflow boundary conditions imposed to represent the vascular bed downstream of the modeled domain. The most common outflow boundary conditions for three dimensional simulations of blood flow, are prescribed constant pressure or traction and prescribed velocity profiles. In many simulations, however, the flow distribution and pressure field in the modeled domain are unknown and cannot be prescribed at the outflow boundaries. The size and complexity of the cardiovascular system necessitate a multidomain approach with models of the upstream regions of interest (large arteries) coupled to reduced-order models of downstream vessels (smaller arteries and microcirculation). A boundary condition was derived using the analytical solution of the linear one-dimensional equations through Green's functions and results showed minimal artificial wave reflection. Alternative downstream models were compared by solving idealized and patient-specific problems. The coupled multidomain method was then extended to the three-dimensional setting following a similar approach and development. A new convective variational formulation was implemented into a stabilized three-dimensional finite element method and analyzed the limitations and opportunities to improve boundary conditions for cardiovascular flow modeling [29].

Misra et al. (2008) developed a mathematical model with an aim to study the pulsatile flow of blood through an arterial segment having a time-dependent stenosis where blood is considered to consist of a core layer where erythrocytes are concentrated and a peripheral plasma layer that is free from erythrocytes. The plasma layer is taken to behave as a Newtonian fluid while the core layer is represented by as a Casson fluid (non-Newtonian) model [13].

Kyoung et al. (2008) investigated the effect of a periodic acceleration and rotating effects in a stenosed blood vessel. The blood flow and wall shear stress are changed under body movement or acceleration variation [10].

Tashtoush et al. (2008) considered heat and fluid flow characteristics of blood flow in multi-stenosis artery in the presence of magnetic field [26].

Varshney and Katiyar (2008) developed a mathematical model for the pulsatile flow of blood through a stenosed artery under the influence of externally imposed body acceleration by using a suitable finite difference scheme in order to obtain the velocity, fluid acceleration, wall shear stress and flow rate [28].

Varshney et al. (2010) proposed a mathematical model for the blood flow in stenosed artery in the presence of transverse magnetic field and effects of overlapping stenosis and externally applied magnetic field is discussed on flow characteristics considering blood as laminar, incompressible and non-Newtonian fluid [27].

Sankar et al. (2010) developed a computational model for analyzing the unsteady flow of blood through stenosed tapered narrow arteries treating blood as a two fluid model with suspension of erythrocytes in the core region as Herschel-Bulkley fluid and the plasma in the peripheral layer as Newtonian fluid [21].

Gupta.S et al. (2011) investigated the effects of stenosis and radial variation of viscosity on the flow characteristics of blood considering laminar, incompressible and non-Newtonian flow of blood using Power-Law fluid model [7].

Jain.N et al. (2012) observed various flow characteristics of blood and effect of parameters of stenosis using Herschel-Bulkley non-Newtonian fluid model considering steady, laminar, one-dimensional flow of blood through an axially non-symmetric but the radically symmetric atherosclerotic artery [8].

Musad (2013) developed a mathematical model for the computation of pressure gradient, viscosity, yield stress and wall shear stress and the influence of stenosis on the rheology of blood where blood flow is assumed to behave like a couple stress fluid, peripheral layer plasma (Newtonian fluid) and core layer of suspension of erythrocytes (Non-Newtonian fluid). The non-Newtonian fluid in the core region of the artery is assumed as a Herschel-Bulkley fluid [15].

Mathur et al. (2013) developed a mathematical model for studying a non-Newtonian flow of blood through stenosed arterial segment using power law fluid model and observed that hemodynamic behavior of blood flow is influenced by the presence of arterial stenosis and results are displayed graphically for different flow characteristics [12].

Shah (2013) studied the effect of non-Newtonian behavior of blood flow through a radially non-symmetric multiple stenosis artery using Herschel-Bulkley fluid model and provided the results for the resistance to flow, apparent viscosity and the wall shear stress through graphical representations [22].

Mallik et al. (2013) developed a mathematical model for studying blood flow through an atherosclerotic artery with slip velocity at wall. A power law fluid model of the blood has been utilized to account for the presence of red cells (erythrocytes) in plasma and quantitative analysis is done on the flow velocity, the volumetric flow rate, the impedance, the wall shear stress and the effective viscosity [11].

Sinha et al. (2013) investigated the influence of externally imposed periodic body acceleration on the flow of blood through a time-dependent stenosed arterial segment by taking into account the slip velocity at the wall of the artery and developed a mathematical model by treating blood as a non-Newtonian fluid obeying casson-fluid model and analytical expressions for various flow characteristics are obtained [24].

Nallapu et al. (2014) studied a two fluid model of Herschel-Bulkley fluid flow through tubes of small diameters and assumed that the core region consists of Herschel-Bulkley fluid and Newtonian fluid in the peripheral region and derived the analytical solutions for velocity, flow flux, effective viscosity, core hematocrit and mean hematocrit and the effects of various relevant parameters on these flow variables have been studied [16].

Srivastava (2014) investigated the MHD blood flow in a porous inclined stenotic artery under the influence of inclined magnetic field considering blood as electrically conducting Newtonian fluid and derived the analytical expressions for various flow characteristics of blood for different parameters involved [25].

2 Purpose of Study

Mathematical modeling and analysis of stenosed artery is of great use for bio-mathematicians and medical scientists because it involves the study of various characteristics of blood.

Our aim is to investigate the flow mechanics of the blood and to see its effects on stenosed artery which will be beneficial in the field of medical sciences as the medical scientists do not have precise information about various flow parameters of blood.

This study becomes highly complex without the use of models owing to the complexity of the analytical solutions of these flow problems, so it is proposed to analyze some physiological problems by using numerical techniques which may be useful for medical doctors.

The long term application of the research to be done is that it promises to be an effective tool for gaining insight into the pathology of arterial diseases. The information to be obtained from this study can be used for better understanding the blood circulation in human body and in the development of new diagnostic tools.

3 Proposed Work

In view of various above problems, some of the following types of problems are proposed to be done:

- Effect of Axial Viscosity Variation of Non-Newtonian Blood Flow in Stenosed Artery.
- Effect of Radial Viscosity Variation of Non-Newtonian Blood Flow in Stenosed Artery.
- Effect of Heat Transfer on Blood Flow through Stenosed Artery.
- Effect of Two-fluid Mathematical Model for blood flow in Stenosed Artery.
- Effect of blood flow through Porous Medium under body acceleration in an artery having Stenosis.
- Impact of Hematocrit Value on viscosity of blood through Stenosed Artery- A Non-Newtonian fluid model.
- Effect of Magnetic Field on blood flow through Catheterized Tapered artery with Mild Stenosis.
- Effect of Bypass in a stenotic artery.

Methodology

In the research work to be done, we will be investigating the behavior of the blood in the stenosed artery using a non- newtonian fluid model. The constitutive equations of the models will be solved analytically using the initial conditions and the given boundary conditions to get expressions for different flow parameters such as flow rate, wall shear stress, resistance to flow etc. and variation of these flow parameters will be analyzed graphically.

Expected Outcome

In the study to be done, the variation of different flow parameters such as flow rate, wall shear stress, resistance to flow will be analyzed. It is expected that the models will generate information about impact of the increase in the stenosis size on the resistance to flow, wall shear stress and flow rate.

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